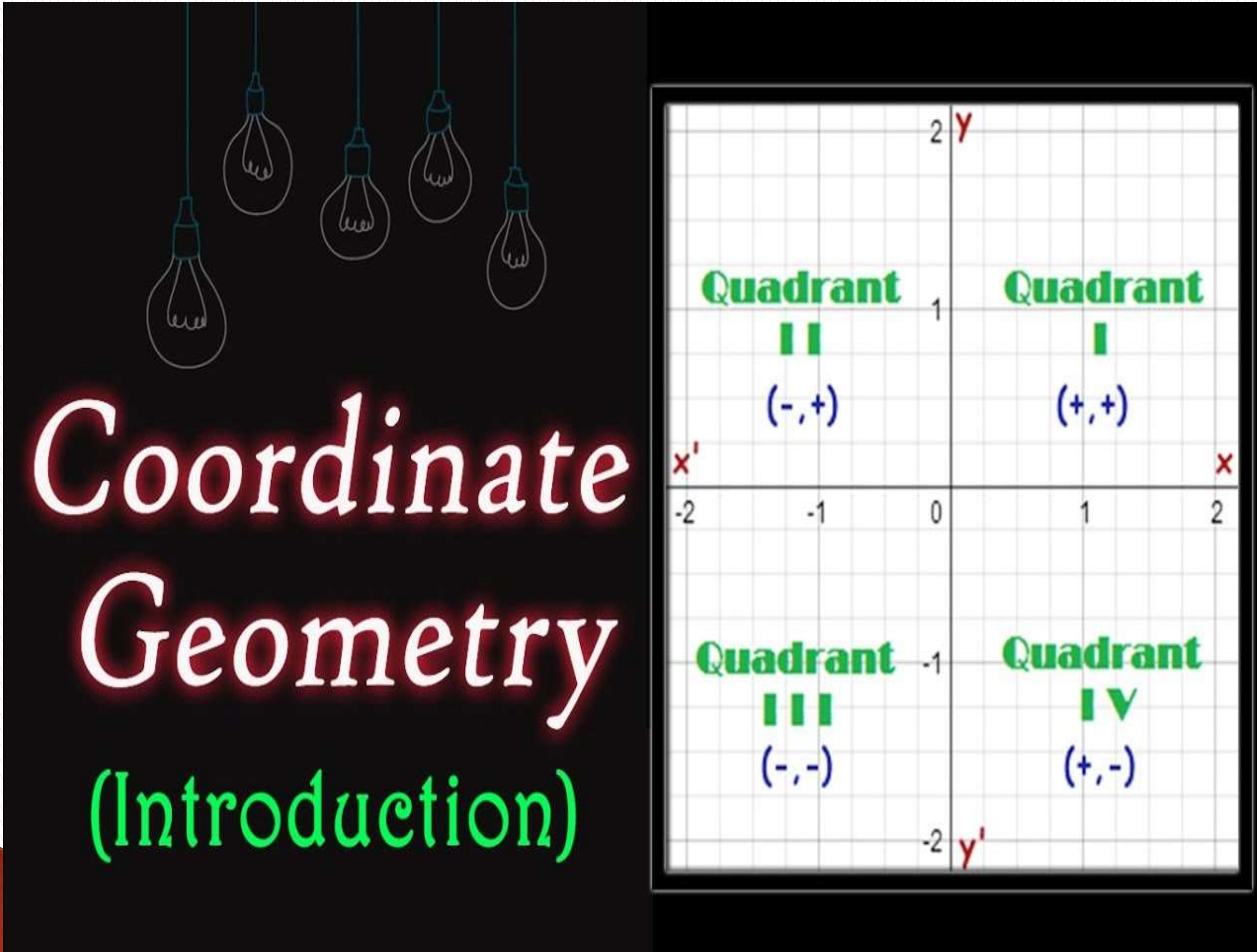
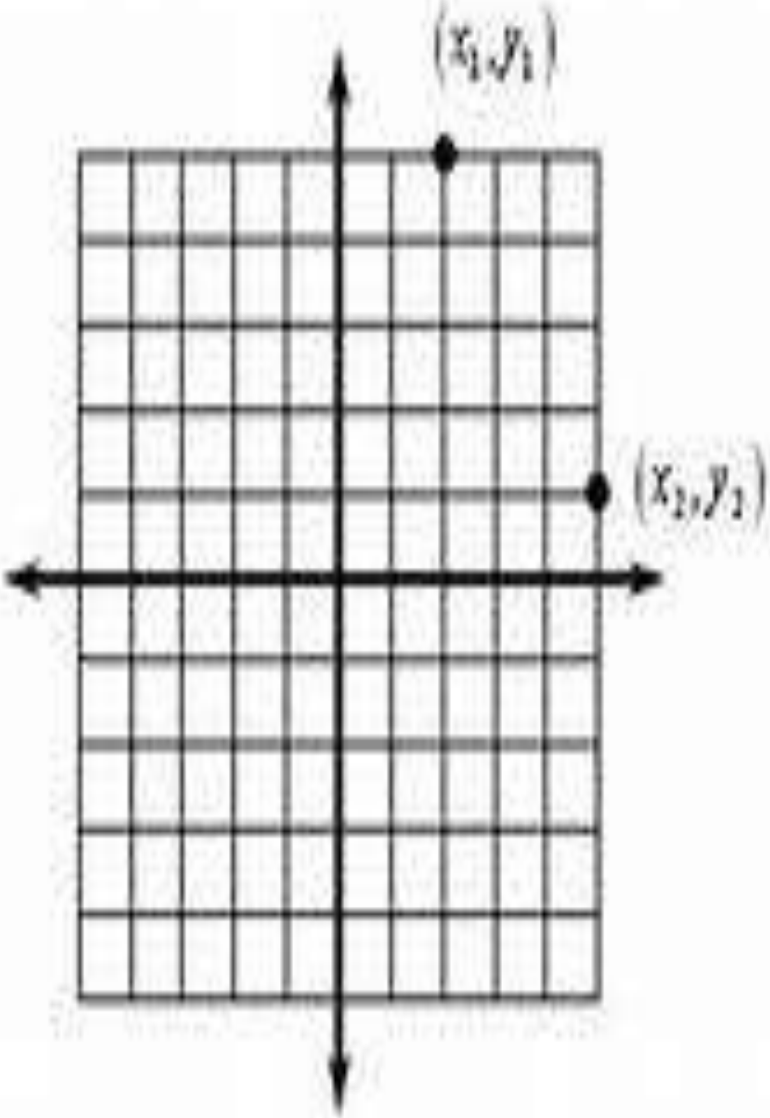
CO-ORDINATE GEOMETRY (TWO DIMENSIONAL)

Coordinate geometry (or analytic geometry) is defined as the study of geometry using the coordinate points.



Coordinate Geometry
(Introduction)
Quadrant II
(-,+)
Quadrant I
(+,+)
Quadrant III
(-,-)
Quadrant IV
(+,-)
x'
x
y
y'
-2
-1
0
1
2



Slope: $m = \dfrac{y_1 - y_2}{x_1 - x_2}$

Midpoint: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

Distance: $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

CO-ORDINATE GEOMETRY

The coordinates of a point on x-axis are of the form $(x, 0)$ and a point on y-axis are of the form $(0, y)$.

Distance Formula

The distance between two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is given by

$$AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

The distance of a point $P(x, y)$ from the origin $O(0, 0)$ is given by

$$OP = \sqrt{x^2 + y^2}$$

Area of Triangle

The area of a triangle with vertices $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ is

$$\frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

$$\frac{1}{2}[(x_1y_2 + x_2y_3 + x_3y_1) - (x_1y_3 + x_2y_1 + x_3y_2)]$$

- In a parallelogram, diagonals bisect each other.
- In a square all four sides are equal and both diagonals are equal.
- In a rectangle opposite sides and both diagonals are equal.

Section Formula

The co-ordinates of the point which divides the join of points $A(x_1, y_1)$ and $B(x_2, y_2)$ internally in the ratio $m : n$ are

$$\left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Mid Point Formula

The coordinates of the mid point of line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

Centroid Formula

The coordinates of centroid of the triangle formed by the points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are

$$\frac{x_1 + x_2 + x_3}{3}, \frac{y_1 + y_2 + y_3}{3}$$

Collinear Points

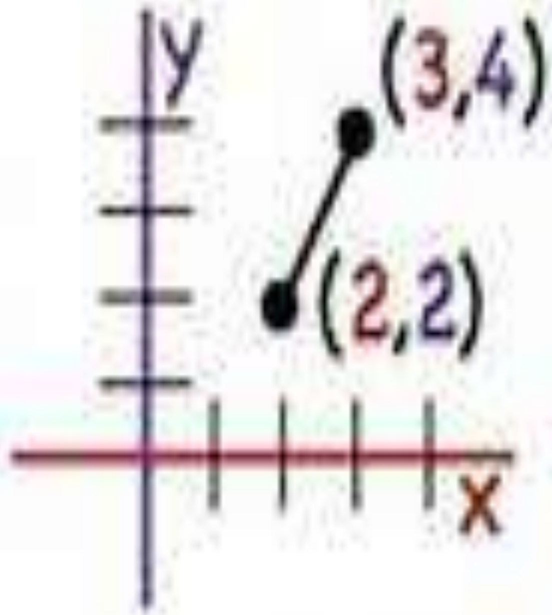
Three points $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are collinear if area of triangle formed by these points is zero.

Three points A, B, C are collinear if $AB + BC = AC$ i.e., sum of distances between two pairs of points is equal to distance between third pair.

Find the Distance

$(3,4)\quad(2,2)$

$\;x_1\;\, y_1 \quad x_2\;\, y_2$



$$d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$$

$$= \sqrt{(2-3)^2+(2-4)^2}$$

$$= \sqrt{(-1)^2+(-2)^2} = \sqrt{1+4} = \sqrt{5} \approx 2.24$$

- Distance formula finds the distance between two points
- Mid-point formula finds the point that is halfway between two points

Distance Formula

$$d=\sqrt{(x-x_1)^2+(y-y_1)^2}$$

Example, find the distance between (2,4) and (5,10)

$$d=\sqrt{(\underset{2}{x}-\underset{5}{x_1})^2+(\underset{4}{y}-\underset{10}{y_1})^2}$$

$$d=\sqrt{(2-5)^2+(4-10)^2}$$

$$=\sqrt{(-3)^2+(-6)^2}$$

$$=\sqrt{9+36}$$

$$d=\sqrt{45}=6.70$$

Find the Distance Between Two Points

Find the distance between $P(-1, 4)$ and $Q(2, -3)$.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$ Distance Formula

$$= \sqrt{[2 - (-1)]^2 + (-3 - 4)^2}$$ Let $(x_1, y_1) = (-1, 4)$ and $(x_2, y_2) = (2, -3)$.

$$= \sqrt{3^2 + (-7)^2}$$ Subtract.

$$= \sqrt{9 + 49} \text{ or } \sqrt{58}$$

Answer: The distance between the two points $\sqrt{58}$ units.

Mid-point formula $\left(\frac{x + x_1}{2}, \frac{y + y_1}{2}\right)$

Example. find the mid-point between

(6, 2) and (2, 10)

$$\left(\frac{x + x_1}{2}, \frac{y + y_1}{2}\right) = \left(\frac{6+2}{2}, \frac{2+10}{2}\right)$$

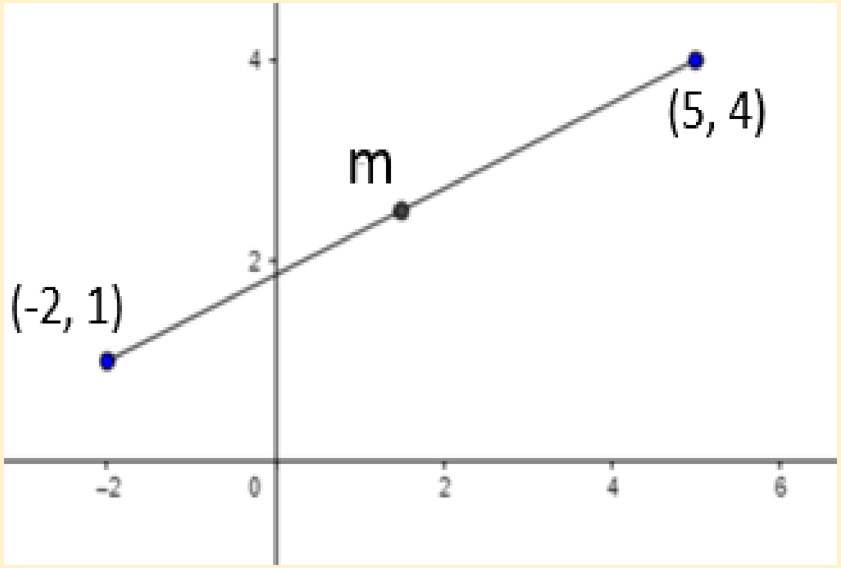
$$= \left(\frac{8}{2}, \frac{12}{2}\right)$$

(4, 6) is halfway between (6, 2), (2, 10)

MP = (4, 6)

Midpoint Formula

$$\text{Midpoint} = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$



$$m = \left(\frac{-2+5}{2}, \frac{1+4}{2}\right)$$

$$= \left(\frac{3}{2}, \frac{5}{2}\right)$$

$$= (1.5, 2.5)$$

Section Formula

So, the coordinates of the point **P**(x, y) which divides the line segment joining the points **A**(x_1, y_1) and **B**(x_2, y_2), internally, in the ratio **m1: m2** are

$$\left(\frac{m_1x_2+m_2x_1}{m_2+m_1},\frac{m_1y_2+m_2y_1}{m_2+m_1}\right)$$

This is known as the **section formula**.

Example In what ratio does the point (– 4, 6) divide the line segment joining the points A(– 6, 10) and B(3, – 8)?

Solution : Let (– 4, 6) divide AB internally in the ratio $m_1 : m_2$. Using the section formula, we get

$$(-4, 6) = \left(\frac{3m_1 - 6m_2}{m_1 + m_2}, \frac{-8m_1 + 10m_2}{m_1 + m_2}\right) \qquad (1)$$

Recall that if $(x, y) = (a, b)$ then $x = a$ and $y = b$.

So, $$-4 = \frac{3m_1 - 6m_2}{m_1 + m_2} \text{ and } 6 = \frac{-8m_1 + 10m_2}{m_1 + m_2}$$

Now, $$-4 = \frac{3m_1 - 6m_2}{m_1 + m_2} \quad \text{gives us}$$

$$-4m_1 - 4m_2 = 3m_1 - 6m_2$$

i.e., $$7m_1 = 2m_2$$

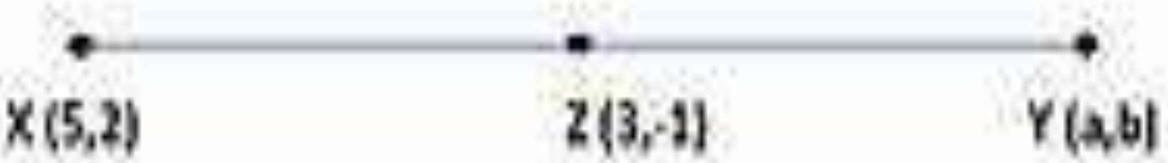
i.e., $$m_1 : m_2 = 2 : 7$$

You should verify that the ratio satisfies the y-coordinate also.

Now, $$\frac{-8m_1 + 10m_2}{m_1 + m_2} = \frac{-8\frac{m_1}{m_2} + 10}{\frac{m_1}{m_2} + 1} \quad \text{(Dividing throughout by } m_2\text{)}$$

$$= \frac{-8 \times \frac{2}{7} + 10}{\frac{2}{7} + 1} = 6$$

Example: Given Z(3,-1) is the midpoint of line X(5,2) & Y(a,b). Find values of a & b.



Using Section Formula:

$$\frac{5+a}{2} = 3 \quad \& \quad \frac{2+b}{2} = -1$$

a= 1 & b=-4 → Answer

Example → Without using distance formula, show that the points (-2,-1) (4,0) (3,3) & (-3,2) are the vertices of a parallelogram.

Solution → Let $A(x_1,y_1) = A(-2,-1)$, $B(x_2,y_2) = B(4,0)$, $C(x_3,y_3) = C(3,3)$, $D(x_4,y_4) = D(-3,2)$

Mid points of AC $= \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

$$= \left(\frac{-2+3}{2}, \frac{-1+3}{2}\right) = \left(\frac{1}{2}, 1\right) \quad \text{(i)}$$

Mid points of BD $= \left(\frac{x_2+x_4}{2}, \frac{y_2+y_4}{2}\right)$

$$= \left(\frac{4-3}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right) \quad \text{(ii)}$$

From equations (i) & (ii)

Mid points of AC = Mid points of BD

∴ the points A, B, C, D are vertices of a parallelogram.

Area of Triangle

If $A(x_1, y_1)$, $B(x_2, y_2)$ & $C(x_3, y_3)$ are the vertices of a ΔABC

then Area of ΔABC

$$= \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$$

Example – Find the area of ΔABC, whose vertices are $A(6,3)$, $B(-3,5)$ & $C(4,-2)$

Sol:- Area of $\Delta ABC = \frac{1}{2}\left[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right]$

Here $A(x_1, y_1) = A(6,3)$

$B(x_2, y_2) = B(-3,5)$

$C(x_3, y_3) = C(4,-2)$

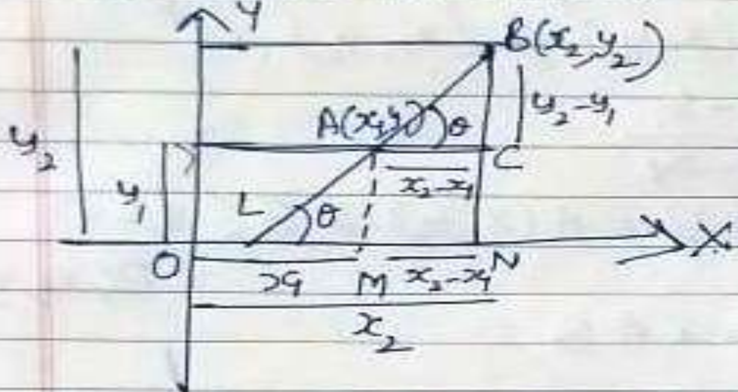
Area of $\Delta ABC = \frac{1}{2}\left[6(5+2) - 3(-2-3) + 4(3-5)\right]$

$= \frac{1}{2}\left[42 + 15 - 8\right]$

$= \frac{49}{2}$ sq. units

Slope of a line joining two points

Let $A(x_1, y_1)$ & $B(x_2, y_2)$ be two given points & θ be the inclination of the line AB, so that $m = \tan\theta$



From A & B draw AM & BN perpendicular to the axis & draw $AC \perp BN$. Let BA meets OX at L

$\angle CAB = \angle XLB = \theta$ (corresponding angles)

$AC = MN = ON - OM = x_2 - x_1$

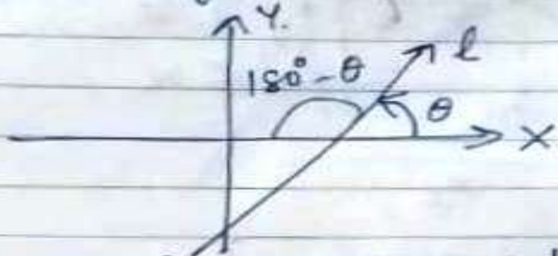
$BC = BN - CN = BN - AM = y_2 - y_1$

In ΔACB, we have $\tan\theta = \frac{BC}{AC}$

$$m = \tan\theta = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope of a straight line

An angle 'θ' made by the line with positive X-axis in anti-clockwise direction is called angle of inclination of a line.
A line in coordinate plane forms two angles with the x-axis, which are supplementary.



$m = \tan\theta$

Note:- (i) When $\theta = 0°$, then line is parallel to x-axis (horizontal line)

(ii) When $\theta = 90°$, then line is perpendicular to x-axis (i.e. ∥ to Y-axis) (vertical line)

Example:- Find the slope of line, whose inclination is 60° & 150°

Sol: Let θ be the inclination of a line, then its slope $= \tan\theta$

at $\theta = 60°$, slope of line $= \tan 60° = \sqrt{3}$

At $\theta = 150°$, slope of line $= \tan 150° = \tan(180° - 30°)$

$= -\tan 30° = -\frac{1}{\sqrt{3}}$

$(\because \tan(180° - \theta) = -\tan\theta)$

Example - Find the slope of a line joining following two points
(i) $A(1,2)$ & $B(3,-4)$ (ii) $(3,-2)$ & $(7,-2)$

Sol. - We know that slope of a line joining two points (x_1,y_1) & (x_2,y_2) is

$$m = \frac{y_2-y_1}{x_2-x_1}$$

(i) Given, $A(1,2) = A(x_1,y_1)$ & $B(3,-4) = B(x_2,y_2)$

$\therefore$ Slope of line AB :

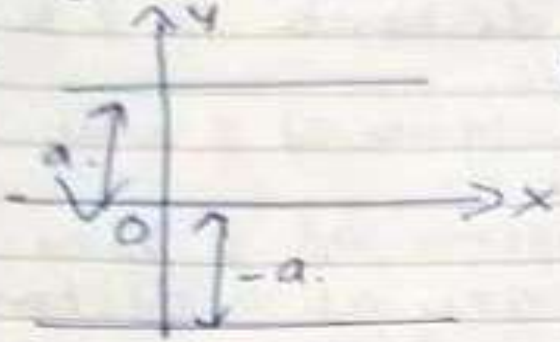
$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-4-2}{3-1} = \frac{-6}{2} = -3$$

(ii) Given, $A(3,-2) = A(x_1,y_1)$ & $B(7,-2) = B(x_2,y_2)$

Slope of line AB :

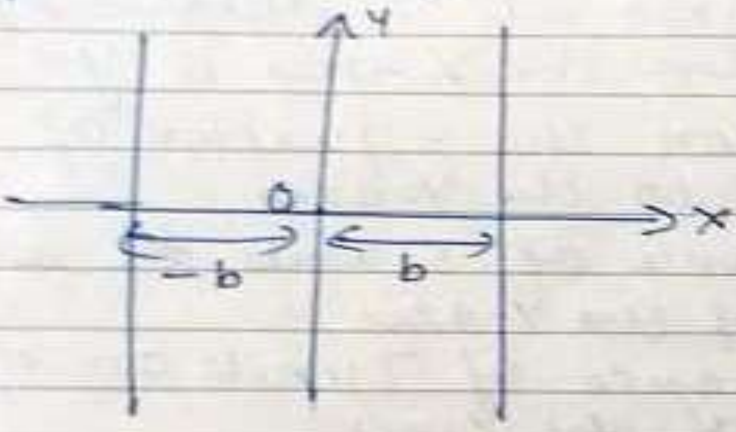
$$m = \frac{y_2-y_1}{x_2-x_1} = \frac{-2+2}{7-3} = \frac{0}{4} = 0$$

Equation of a line ∥ to x-axis (or equation of horizontal line)



$y = a$ or $y = -a$

Equation of a line ∥ to y-axis (or equation of vertical line)



$x = b$ or $x = -b$

Note:- Equation of x-axis is $y = 0$

Equation of y-axis is $x = 0$

Example → Write down the equation of a line parallel to the X-axis
(i) at a distance of 6 units above the X-axis.
(ii) at a distance of 3 units below the X-axis

Solution: (i) The equation of a line parallel to the X-axis at a distance of 6 units above X-axis is $y = 6$

(ii) The equation of a line parallel to the X-axis at a distance of 3 units below the X-axis is $y = -3$

Example. Write down the equation of a line parallel to the Y-axis
(i) at a distance of 5 units on left hand side of the Y-axis
(ii) at a distance of 7 units on right hand side of the Y axis.

Solution → (i) The equation of a line parallel to the Y-axis at a distance of 5 units on its left is $x = -5$

(ii) The equation of a line parallel to

the Y-axis at a distance of 7 units on its right is $x = 7$

Point Slope Form :-

The equation of the straight line having slope m and passes through the pt $Q(x_1, y_1)$ is : $y - y_1 = m(x - x_1)$

Example :- Find the equation of the line passing through $(-4, 3)$ & having slope $\frac{1}{2}$.

Solution → Equation of the line passing through the point (x_1, y_1) and having slope 'm' is $y - y_1 = m(x - x_1) \longrightarrow$ (i)

m = slope of the line $= \frac{1}{2}$

$x_1 = -4$, $y = 3$

∴ The required equation of the line is

$$y - 3 = \frac{1}{2}(x + 4)$$

$$2y - 6 = x + 4$$

$$x - 2y + 10 = 0$$

Example:- Find the equation of the perpendicular bisector of the line segment joining the points A(2,3) & B(6,-5)

Solution:- The slope of AB is given by

$$m = \frac{-5-3}{6-2} = -2 \quad \left\{ \because m = \frac{y_2-y_1}{x_2-x_1} \right\}$$

$\therefore$ Slope of a line perpendicular to AB $= -1/m = 1/2$

Let P be the mid point of AB

Then the coordinates of P are

$$\left(\frac{2+6}{2}, \frac{3-5}{2} \right) \text{ i.e. } (4,-1)$$

Thus, the required line passes through P(4,-1) & has slope $\frac{1}{2}$, so its equation is

$$y+1 = \frac{1}{2}(x-4) \quad \left(\because y - y_1 = m(x-x_1) \right)$$

$$2y+2 = x-4$$

$$x-2y-6=0$$

Example → Two lines passing through the point (2, 3) intersect each other at an angle 60°. If slope of one line is 2, then find the equation of the other line.

Sol:- Let the slope of the other line be m.

It is given that angle between two lines is 60°.

$$\therefore \tan 60^\circ = \left|\frac{m-2}{1+2m}\right| \qquad \left[\because \tan\theta = \left|\frac{m_2-m_1}{1+m_1 m_2}\right|\right]$$

$$\Rightarrow \sqrt{3} = \left|\frac{m-2}{1+2m}\right| \Rightarrow \frac{m-2}{1+2m} = \pm\sqrt{3}$$

$$\Rightarrow \frac{m-2}{1+2m} = \sqrt{3} \quad \text{or} \quad \frac{m-2}{1+2m} = -\sqrt{3}$$

$$\Rightarrow m-2 = \sqrt{3}+2\sqrt{3}m \quad \text{or} \quad m-2 = -\sqrt{3}-2\sqrt{3}m$$

$$\Rightarrow m(2\sqrt{3}-1) = -(2+\sqrt{3}) \quad \text{or} \quad m(2\sqrt{3}+1) = 2-\sqrt{3}$$

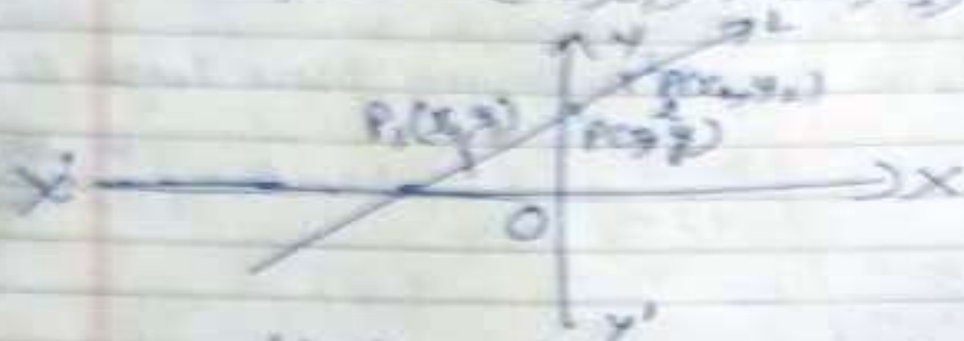
$$\Rightarrow m = -\left(\frac{2+\sqrt{3}}{2\sqrt{3}-1}\right) \quad \text{or} \quad m = \frac{2-\sqrt{3}}{2\sqrt{3}+1}$$

On substituting $x_1 = 2$, $y_1 = 3$ and the value of 'm' in $y - y_1 = m(x - x_1)$, we obtain that the equation of the required line is

$$y-3 = -\left(\frac{2+\sqrt{3}}{2\sqrt{3}-1}\right)(x-2) \quad \text{or} \quad y-3 = \frac{2-\sqrt{3}}{2\sqrt{3}+1}(x-2)$$

Two Point Form

The equation of a line passing through the points (x_1, y_1) & (x_2, y_2) is given by



$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Example :- Find the equation of the straight line passing through the points $(-1, 1)$ & $(2, -4)$

Solution :- Let the given points are $A(x_1, y_1) = A(-1, 1)$ & $B(x_2, y_2) = B(2, -4)$

$$y - 1 = \frac{-4 - 1}{2 + 1}(x + 1) \qquad \left[\because y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)\right]$$

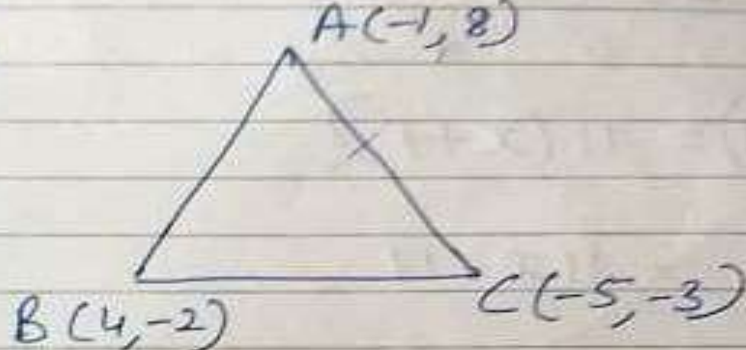
$$y - 1 = \frac{-5}{3}(x + 1)$$

$$3y - 3 = -5x - 5$$

$$5x + 3y + 2 = 0$$

Example:- Find the equations of the sides of a triangle whose vertices are A(-1, 8), B(4, -2) & C(-5, -3)

Solution →



Equation of AB is : $y - 8 = \frac{-2-8}{4+1}(x+1)$

$\Rightarrow y - 8 = -2x - 2$

$\Rightarrow 2x + y - 6 = 0$

Equation of BC is :

$\Rightarrow y + 2 = \frac{-3+2}{-5-4}(x-4)$

$\Rightarrow y + 2 = \frac{\cancel{-}1}{\cancel{-}9}(x-4)$

$\Rightarrow 9y + 18 = x - 4$

$\Rightarrow x - 9y - 22 = 0$

Equation of AC is

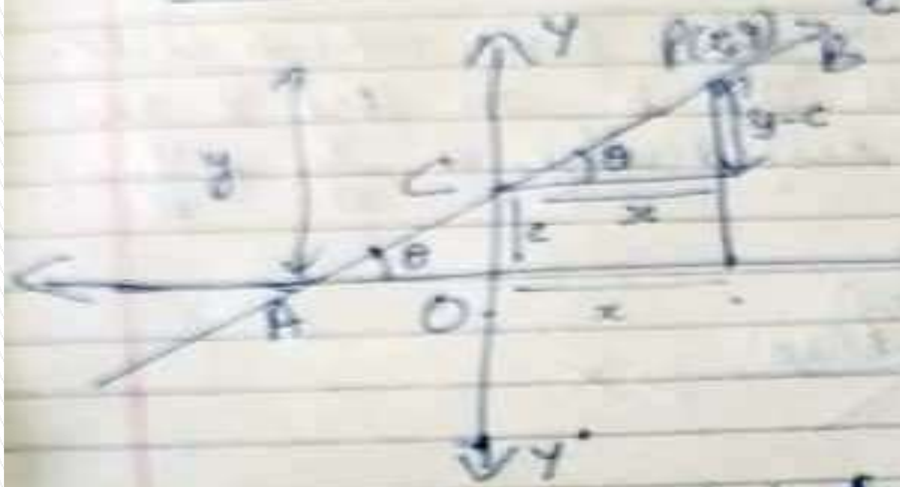
$$\Rightarrow y-8 = \frac{-3-8}{-5+1}(x+1)$$

$$\Rightarrow 4(y-8) = 11(x+1)$$

$$\Rightarrow 4y-32 = 11x+11$$

$$\Rightarrow 11x-4y+43=0$$

Slope intercept form



Suppose a line L with slope m cuts the y-axis at a distance c from the origin. (distance c is called y-intercept of the line)

Equation of line L

$$y = mx + c$$

Example:- Find the equation of the line which have slope $\frac{1}{2}$ and cuts off an intercept -5 on y-axis

Solution:- Here c = intercept of the line on y-axis $= -5$

$$m = \frac{1}{2}$$

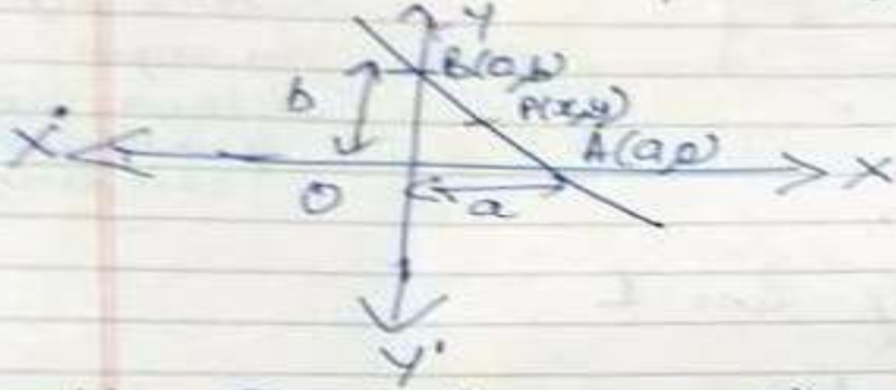
$$y = mx + c$$

$$y = \frac{1}{2}x - 5 \quad ; \quad 2y = x - 10$$

$$x - 2y - 10 = 0$$

Intercept form

The equation of a line which cuts-off intercepts 'a' & 'b' on the X-axis and Y-axis respectively, is $\frac{x}{a} + \frac{y}{b} = 1$



ample:- Find the equation of a line having intercepts '−2' & '3'.

lution:- The given intercepts are $a=-2$, $b=3$

Equation of line in intercept form

$$\frac{x}{a} + \frac{y}{b} = 1$$

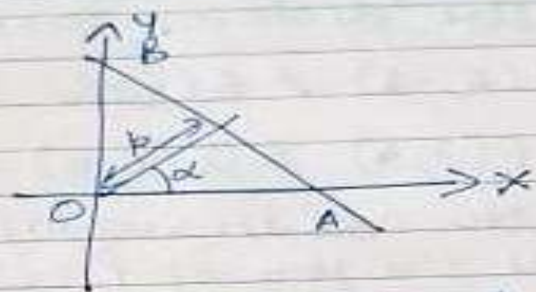
$$\frac{x}{-2} + \frac{y}{3} = 1$$

$$\frac{-3x + 2y}{6} = 1$$

$$-3x + 2y = 6$$

$$3x - 2y + 6 = 0$$

Normal Form or Perpendicular Form of a line:-



The equation of the straight line upon which the length of perpendicular (i.e. normal) from the origin is 'p' and this perpendicular (i.e. normal) makes an angle α with the positive direction of X-axis is:-

$$x\cos\alpha + y\sin\alpha = p$$

Example:- Find the equation of the line upon which the length of perpendicular 'p' from origin and the angle α made by this perpendicular with the positive direction of X-axis are $p=4$, $\alpha=120^\circ$

Sol:- The equation of line, having perpendicular distan 'p' from the origin and angle 'α' which the ⊥ makes with the +ve direction of X-axis is

$x\cos\alpha + y\sin\alpha = p$

$p=4,\ \alpha=120^\circ$

$x\cos 120^\circ + y\sin 120^\circ = 4$

$x\cos(180^\circ-60^\circ) + y\sin(180^\circ-60^\circ) = 4$

$-x\cos 60^\circ + y\sin 60^\circ = 4$

$-\dfrac{x}{2} + \dfrac{\sqrt{3}\,y}{2} = 4$

$-x + \sqrt{3}\,y = 8$

Assignment

Q-1 Find the distance between the points

(i) $A(2,-3)$ & $B(6,-3)$

(ii) $P(3,4)$ & $Q(0,0)$

Q-2 Find the coordinates of the point which divides the join of $P(-5,11)$ & $Q(4,-7)$ in the ratio $2:7$.

Q-3 Find the area of ΔABC where $A(2,-1)$, $B(3,2)$, $C(5,1)$

Q-4. Find the equation of the line through $(-2,3)$ with slope -4.

Q-5 Find the equation of line passing through $(0,2)$ & $(3,-3)$

Q-6 Find the equation of a line, which passes through the point $(2,3)$ and makes an angle of 30° with the positive direction of X-axis.

Q-7 Find the equation of straight line which passes through the point (5, 6) and has intercepts on the axes equal in magnitude but opposite in sign.

Q-8 Find the equation of the line passing through the point (5, 2) & perpendicular to the line joining the points (2, 3) & (3, -1)

Q-9 Find the equation of the line, whose length of the perpendicular distance from the origin is 5 units and the angle made by the perpendicular with positive x-axis is 30°

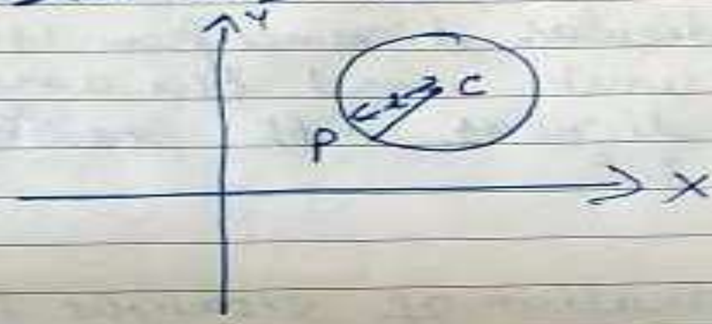
Q-10 Find the equation of straight line which passes through (3, 4) & the sum of whose intercepts on the coordinate axes is 14.

Circle

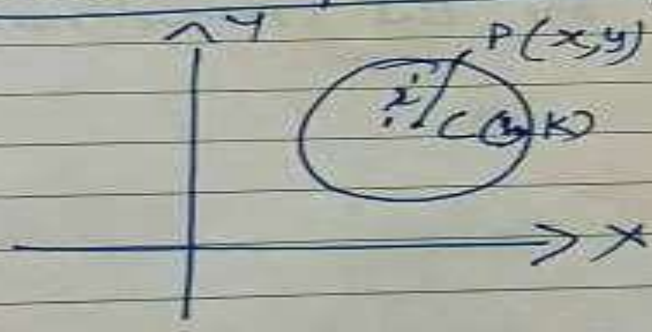
Definition:- A circle is the set of all point in a plane that are equidistant from a fixed point in that plane.

Centre:- The fixed point C is called the centre of the circle.

Radius:- The fixed distance CP from the centre C to a point on the circle say P is called the radius = CP = r



Standard equation of circle.



Let C(h,K) be the centre of the circle, P(x,y) be any point on the circle and r be the radius

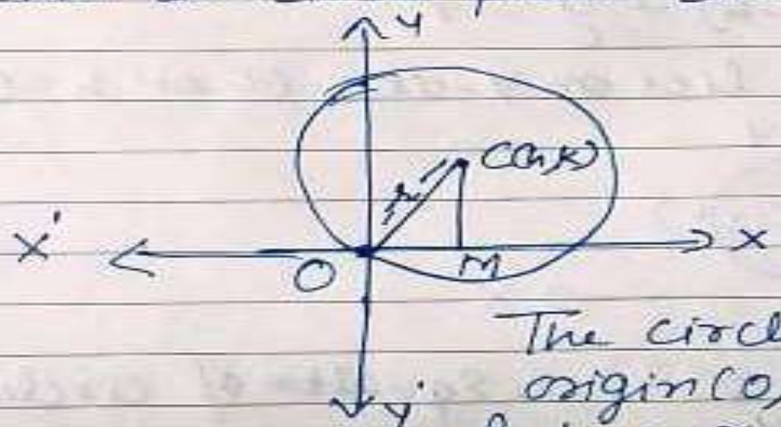
Then, equation of circle in standard form is

$$(x-h)^2+(y-k)^2=r^2$$

Note :- If $C(h,k) = C(0,0)$
then equation of circle is $x^2+y^2=r^2$

Different Types of standard equation of circle

① When the circle passes through the origin



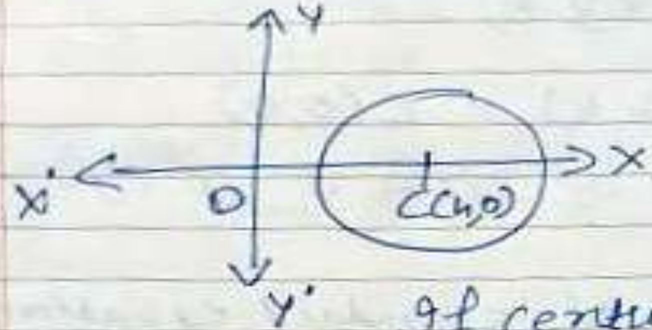
The circle passes through origin $(0,0)$
Radius $= r =$ Distance between points O & C

$$= \sqrt{h^2+k^2}$$

Equation of circle having centre (h,k) & $r=\sqrt{h^2+k^2}$

$$(x-h)^2+(y-k)^2=\left(\sqrt{h^2+k^2}\right)^2=h^2+k^2$$

(2) When the centre lies on X-axis or Y-axis

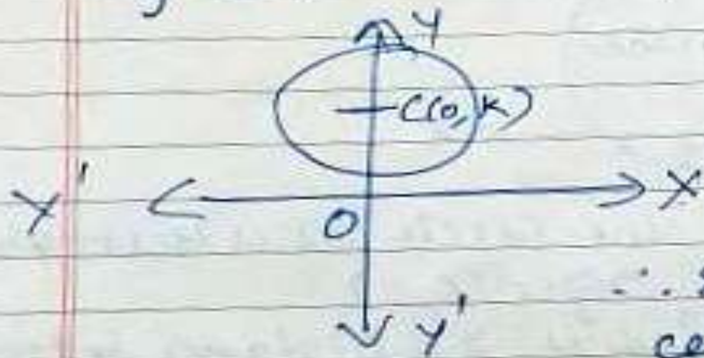


If centre lies on X-axis then $k=0$.

$\therefore$ Equation of circle with centre $C(h,0)$ is

$$(x-h)^2+y^2=r^2$$

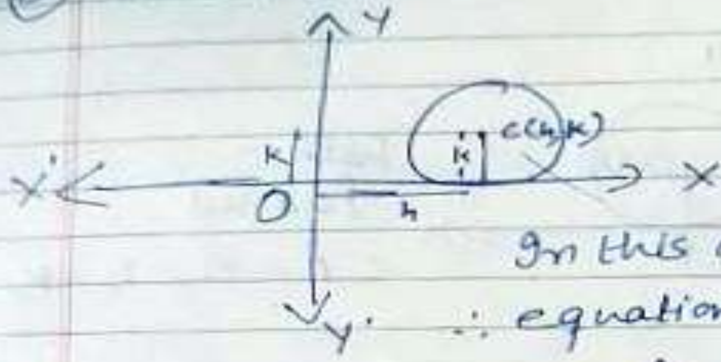
If centre lies on Y-axis then $h=0$



$\therefore$ Equation of circle with centre $C(0,k)$ is

$$x^2+(y-k)^2=r^2$$

(3) When circle touches X-axis

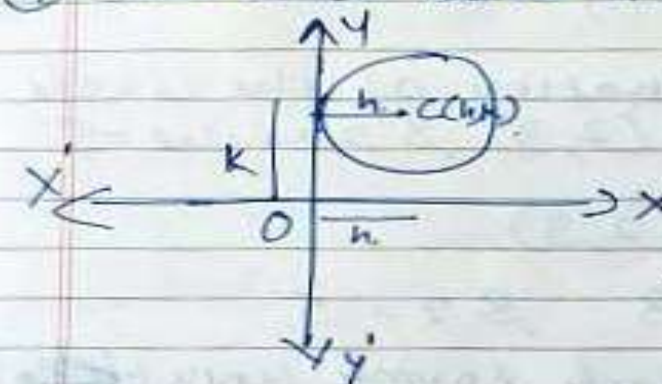


In this case $r = K$

$\therefore$ equation of circle is :-

$$(x-h)^2 + (y-K)^2 = K^2$$

(4) When circle touches Y-axis

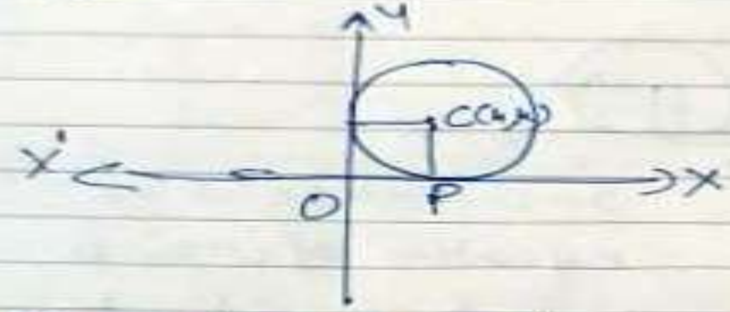


In this case $r = h$

$\therefore$ equation of circle is

$$(x-h)^2 + (y-K)^2 = h^2$$

(5) When the circle touches both the axes :-



Radius
$CP = r = h = K$

Equation of circle :-

$$(x-h)^2 + (y-h)^2 = h^2$$

Example :- Find the equation of the circle with centre = (2,3) & radius = 5

Solution :- Centre = (2,3)

$h = 2, K = 3 \quad \& \; r = 5$

Equation of circle having centre (h,k)

$$(x-h)^2 + (y-k)^2 = r^2$$

$$(x-2)^2 + (y-3) = (5)^2$$

$$x^2 + 4 - 4x + y^2 + 9 - 6y = 25$$

$$x^2 + y^2 - 4x - 6y - 12 = 0$$

Example → Find the equation of circle whose centre is (1, 2) & which passes through the point (4, 6).

Solution → Coordinates of centre of given circle is (1,2) & it passes through the point (4, 6).

Then radius of the circle is equal to the distance from the centre to a point on a circle.

$$\therefore \text{Radius of circle} = \sqrt{(1-4)^2+(2-6)^2}$$

$$= \sqrt{9+16} = 5$$

Now the equation of circle having centre (1,2) & radius = 5

$$(x-1)^2+(y-2)^2=(5)^2$$

$$x^2+1-2x+y^2+4-4y=25$$

$$x^2+y^2-2x-4y-20=0$$

Example → Find the equation of circle whose centre is $(1,2)$ and touches x-axis.

Sol:- Given, centre $(h,k) = (1,2)$
and circle touches on x-axis.
Radius (r) = y-coordinate of centre $= 2$

$\therefore$ equation of circle is:-

$$(x-1)^2+(y-2)^2=(2)^2 \quad \left[\because (x-h)^2+(y-k)^2=r^2\right]$$

$$x^2+y^2-2x-4y+1=0$$

Example → Find the equation of the circle which touches both the axes and whose radius $= 5$

Solution → As the circle touches both the axes so $h=k=r$

i.e. $h=k=r=5$

$\therefore$ Reqd. equation of circle is

$$(x-5)^2+(y-5)^2=(5)^2$$

$$x^2+y^2-10x-10y+25=0$$

General Equation of a circle :-

Since the standard form of circle is

$$(x-h)^2+(y-k)^2=r^2$$

$$x^2+y^2-2hx-2yk+h^2+k^2-r^2=0$$

which is of the form

$$x^2+y^2+2gx+2fy+c=0$$

On comparing $g=-h$, $\cancel{g}$ $f=-k$

$$c=h^2+k^2-r^2$$

Then the above equation of circle is known as general equation with

Centre $(-g,-f) = C\left(-\frac{1}{2}(\text{coeff of } x), -\frac{1}{2}(\text{coeff of } y)\right)$

& Radius $r=\sqrt{h^2+k^2-c}$

or $r=\sqrt{g^2+f^2-c}$

Note: (1) Coeff of x^2 = Coeff of y^2

(2) There should not be mixed term like 'xy'

Example Find the centre and radius of each of the following circles:

(i) $x^2+(y+2)^2 = 9$ (ii) $x^2+y^2+6x-4y+4=0$

(iii) $3x^2+3y^2 = 27$ (iv) $x^2+y^2-6x+5y-8=0$

Solution: (i) Given equation of circle is

$$x^2+(y+2)^2 = 9$$

$$(x-0)^2+(y-(-2))^2 = (3)^2$$

On comparing with $(x-h)^2+(y-k)^2 = r^2$

$h=0,\ k=-2,\ r=3$

$C(0,-2),\ r=3$

(ii) The given equation of circle is :-

$x^2+y^2+6x-4y+4=0$

Comparing it with $x^2+y^2+2gx+2fy+c=0$

$2g=6,\ 2f=-4,\ c=4$

$g=3,\ f=-2,\ c=4$

$C(-g,-f)=C(-3,2)$

$r=\sqrt{g^2+f^2-c}=\sqrt{9+4-4}=\sqrt{9}=3$

(iii) $3x^2+3y^2=27$

Dividing both sides with '3'

$x^2+y^2-9=0$

Comparing it with $x^2+y^2+2gx+2fy+c=0$

$2g=0,\ 2f=0,\ c=-9$

$g=0,\ f=0$

$C(-g,-f)=C(0,0)$

$r=\sqrt{g^2+f^2-c}=\sqrt{0+0+9}=3$

(iv) The given equation is :-

$$x^2 + y^2 - 6x + 5y - 8 = 0$$

Comparing it with :-

$$x^2 + y^2 + 2gx + 2fy + c = 0$$

$$2g = -6, \quad 2f = 5, \quad c = -8$$

$$g = -3, \quad f = 5/2, \quad c = -8$$

Centre :- $C(-g, -f)$

$$= C(3, -5/2)$$

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{9 + \frac{25}{4} + 8}$$

$$= \sqrt{\frac{68 + 25}{4}} = \frac{\sqrt{93}}{2}$$

Example → Find the equation of the circle passing through the point (2,4) and having its centre at the intersection of the lines $x-y=4$ and $2x+3y=-7$.

Solution →

The given lines are $x-y=4$ and $2x+3y=-7$

Solving these two equations we get $x=1$, $y=-3$

The point of intersection of given lines is the centre $C(1,-3)$

$$CP = r = \sqrt{(2-1)^2+(4+3)^2}$$

$$= \sqrt{1+49} = \sqrt{50}$$

Hence, the required equation of circle whose centre is $C(1,-3)$ and radius $= \sqrt{50}$

$$(x-1)^2+(y+3)^2 = (\sqrt{50})^2 \quad \left(\because (x-h)^2+(y-k)^2 = r^2\right)$$

$$x^2+1-2x+y^2+9+6y = 50$$

$$x^2+y^2-2x+6y-40=0$$

Example:- Find the equation of the circle passing through the points $(2,-6)$, $(6,4)$ & $(-3,1)$

Solution:- let equation of the circle passing through the points is

$$x^2+y^2+2gx+2fy+c=0 \longrightarrow (1)$$

Since the circle passes through the point $(2,-6)$

$$(2)^2+(-6)^2+2g(2)+2f(-6)+c=0$$

$$4g-12f+c=-40 \longrightarrow (2)$$

The circle passes through $(6,4)$

$$(6)^2+(4)^2+2g(6)+2f(4)+c=0$$

$$12g+8f+c=-52 \longrightarrow (3)$$

Circle passes through $(-3,1)$

$$(-3)^2+(1)^2+2g(-3)+2f(1)+c=0$$

$$-6g+2f+c=-10 \longrightarrow (4)$$

Putting the value of 'g' in (5) we get

$$-2\left(\frac{-32}{13}\right) - 5f = 3$$

$$\frac{64}{13} - 5f = 3$$

$$5f = \frac{64}{13} - 3 = \frac{64-39}{13} = \frac{15}{13}$$

$$f = \frac{5}{13}$$

~~Substituting~~ Substituting the values of g & f in (4) we get

$$-6\left(\frac{-32}{13}\right) + 2\left(\frac{5}{13}\right) + c = -10$$

$$\frac{192}{13} + \frac{10}{13} + c = -10$$

$$c = -10 - \frac{192}{13} - \frac{10}{13}$$

$$= \frac{-130-192-10}{13} = \frac{-332}{13}$$

Substituting the values of g, f & c in equation (i) we get

$$x^2+y^2+2\left(\frac{-32}{13}\right)x+2\left(\frac{5}{13}\right)y-\frac{332}{13}=0$$

$$13x^2+13y^2-64x+10y-332=0$$

Concentric Circles

Two circles having the same centre $C(h,k)$ but different radii r_1 & r_2 are called concentric circles.

Thus, the circles

$$(x-h)^2+(y-k)^2=r_1^2$$

& $(x-h)^2+(y-k)^2=r_2^2$, $r_1 \neq r_2$ are concentric circles.

<u>Example</u> → Find the equation of the circle which passes through the centre of circle: $x^2+y^2+8x+10y-7=0$ and is concentric with the circle $2x^2+2y^2-8x-12y-9=0$

Solutions:- Given circles are $x^2+y^2+8x+10y-7=0 \rightarrow (i)$

& $2x^2+2y^2-8x-12y-9=0 \rightarrow (ii)$

Centre of circle (i) is $C(-4,-5)$

Equation of the circle concentric with circle (ii) is

$$2x^2+2y^2-8x-12y+c=0 \rightarrow (iii)$$

This circle passes through $C(-4,-5)$

$2(-4)^2 + 2(-5)^2 - 8(-4) - 12(-5) + c = 0$

$32 + 50 + 32 + 60 + c = 0$

$c = -174$

Substituting the value of 'c' in equation (ii)

$2x^2 + 2y^2 - 8x - 12y - 174 = 0$

$x^2 + y^2 - 4x - 6y - 87 = 0$

Diametric Form of a circle

Let (x_1, y_1) and (x_2, y_2) be the end points of the diameter of a circle. The equation of the circle drawn through these points is

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Example:- Find the equation of the circle whose end points of a diameter are $A(1,5)$ & $B(-1,3)$

Sol:- Given end points of a diameter are $A(\overset{x_1}{1},\overset{y_1}{5})$ & $B(\overset{x_2}{-1},\overset{y_2}{3})$

Equation of circle in diametric form:-

$$(x-x_1)(x-x_2)+(y-y_1)(y-y_2)=0$$

Thus the reqd. equation is:-

$$(x-1)(x+1)+(y-5)(y-3)=0$$

$$x^2-1+y^2-8y+15=0$$

$$x^2+y^2-8y+14=0$$

FRIDAY

Assignment

Q-1. Find the equation of the circle with

(i) Centre $= (-3, 2)$ & radius $= 5$

(ii) Centre $\left(\frac{1}{3}, \frac{1}{4}\right)$ & radius $= 12$

Q-2. Find the centre and radius of the circle of following :-

(i) $(x+5)^2 + (y-3)^2 = 36$

(ii) $x^2 + y^2 - 6x + 4y - 12 = 0$

(iii) $x^2 + y^2 - 2x + 4y = 0$

(iv) $3x^2 + 3y^2 + 6x - 4y - 1 = 0$

Q-3- Find the equation of the circle whose centre is $(2, -5)$ & which passes through the point $(3, 2)$

Q.4 Find the equation of the circle passing through $(-2, 3)$, $(5, 2)$ & $(6, -1)$

Q-5 Find the equation of the circle in which end points of diameter are $(2, -2)$ & $(3, 4)$.

26 SUNDAY